Name____

Quantum Mechanics
Quiz 2

1D Schrödinger eqn:

$$H\,\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$$

Most convenient form of $\psi(x,t)=$?

a) $X(x)$
b) $T(t)$
c) $X(x)\,T(t)$
d) none

$\psi'' = -k^2\psi \qquad k=\text{real}$

$\psi(x)=$?

a) $Ae^{ikx}+Be^{-ikx}$
b) $A\sin(kx)+B\cos(kx)$
c) $A\sinh(kx)+B\cosh(kx)$
d) none

Suppose
$f(x)=g(t)$ for all x,t
Then:

a) $x=t$
b) $f=g=$ constant
c) $f=g=0$
d) none

Suppose
$\psi(x)=A\sin(kx)$
Apply $\psi(L)=0$. Conclusion?

a) $A=0$
b) $B=0$
c) $k=n\pi/L$
d) none

A plane wave $e^{i(kx-\omega t)}$ is a good model for a real particle

a) T
b) F

Generally, no, but we use it despite some contradictions,

I accepted either answer w/ explanation.

Properties of $\delta(x-x_0)$

$$\delta(x-x_0) = \begin{cases} 0 & x\neq x_0 \\ \infty & x=x_0\end{cases}$$

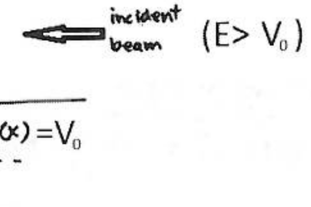
$$\int_{-\infty}^{+\infty}\delta(x-x_0)\,dx = 1$$

$$\int_{-\infty}^{+\infty}f(x)\,\delta(x-x_0)\,dx = f(x_0)$$

Name____

$\frac{6}{6}$

Step potential. A beam of electrons moves towards a step potential as shown. The electrons are *incident from the right*, and will be modeled as plane waves with energy $E>V_0$.



$$Ce^{-ik_I x} \quad \text{I} \quad \vdots \quad \text{II} \quad Ae^{-ik_{II}x} + Be^{+ik_{II}x}$$

1. Write appropriate forms of ψ_I and ψ_{II} in terms of plane waves. You *do not* have to start from the Schroedinger equation. You may simply write down the forms, but you should justify your choices based on the general physics of wave behavior. Explain.

$$\psi_I = C\underbrace{e^{-ik_I x}}_{\text{trans}}$$

$$\psi_{II} = A\underbrace{e^{-ik_{II}x}}_{\text{inc}} + B\underbrace{e^{+ik_{II}x}}_{\text{ref}}$$

Since the wave is incident from the left, region II will have the incident and reflected waves, and I will have the transmitted.

$\frac{2}{2}$

2. Write down what k_I and k_{II} are in terms of the variables given. Explain.

$$k_I = \sqrt{\frac{2m}{\hbar^2}(E-0)}$$

$$k_{II} = \sqrt{\frac{2m}{\hbar^2}(E-V_0)}$$

These come from rewriting the SE $\rightarrow$ $\psi''(x) = -\underbrace{\frac{2m}{\hbar^2}(E-V(x))}_{k^2}\psi(x)$

$\frac{4}{4}$

3. Write down the appropriate matching conditions at x=0, and use these to determine two equations in terms of wave amplitudes A, B, and C. *You do not have to solve these equations, just obtain them.* Explain.

$$\psi_I(0) = \psi_{II}(0) \rightarrow \boxed{C = A + B}$$

$$\psi_I'(0) = \psi_{II}'(0) \rightarrow \boxed{-k_I C = -k_{II}A + k_{II}B}$$

The first is continuity of the wave function at $x=0$

The second is continuity of the derivative, which we expect when $V(x)$ is well-behaved.