Homework #9

4-3.2

Date (1961 0^h E.T.)	$\alpha(1950)$	$\delta(1950)$
Nov. 1.0	$4^h 2^m 1.30^s$	$11° 38' 42.8''$
Nov. 11.0	$3^h 52^m 35.17^s$	$11° 15' 58.7''$
Nov. 27.0	$3^h 35^m 49.42^s$	$10° 51' 18.4''$

at three dates

	X	Y	Z
$\vec{R}_1$	-.7802476	-.5626925	-.2440132
$\vec{R}_2$	-.6598348	-.6771343	-.2936387
$\vec{R}_3$	-.4271833	-.8159774	-.3538531

and I asked you to find $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$. We should be able to leave the time E.T. (UT - 5.0h) for current purposes - especially as we can't interpolate to get $\vec{R}$ and must assume X, Y, Z are at the times of observation.

t_1 = Nov. 1.0, 1961 $\quad \alpha_1 = 60.5054167°$ $\quad \delta_1 = 11.6452222°$
t_2 = Nov. 11.0, 1961 $\quad \alpha_2 = 58.1465417°$ $\quad \delta_2 = 11.2663056°$
t_3 = Nov. 27.0, 1961 $\quad \alpha_3 = 53.9559167°$ $\quad \delta_3 = 10.8551111°$

It is interesting to note that the asteroid is in a retrograde motion here.

$$\hat{u} = \cos\alpha\cos\delta\,\hat{\imath} + \sin\alpha\cos\delta\,\hat{\jmath} + \sin\delta\,\hat{k}$$

$$\hat{u}_1 = .48220704\,\hat{\imath} + .85248609\,\hat{\jmath} + .20185104\,\hat{k}$$
$$\hat{u}_2 = .51757867\,\hat{\imath} + .83303247\,\hat{\jmath} + .19536943\,\hat{k}$$
$$\hat{u}_3 = .57787890\,\hat{\imath} + .79409651\,\hat{\jmath} + .18832606\,\hat{k}$$

$$T_1 = k(t_3 - t_2) = k(16d) = .27523358$$
$$T_2 = T_1 + T_3 = k(t_3 - t_1) = k(26d) = .44725457$$
$$T_3 = k(t_2 - t_1) = k(10d) = .17202099$$

$$a_1 = \frac{T_1}{T_2} = \frac{16}{26} = .61538462 \qquad b_1 = \frac{T_1}{6T_2}(T_2^2 - T_1^2) = .01274699$$
$$a_3 = \frac{T_3}{T_2} = \frac{10}{26} = .38461539 \qquad b_3 = \frac{T_3}{6T_2}(T_2^2 - T_3^2) = .01092599$$

$c_1 = .61538462 + .01274699/r_2^3$

$c_3 = .38461539 + .01092599/r_2^3$

$$\hat{u}_1 \cdot \hat{u}_2 \times \hat{u}_3 = \begin{vmatrix} .48220704 & .85248609 & .20185104 \\ .51757867 & .83303247 & .19536943 \\ .57787890 & .79409651 & .18832606 \end{vmatrix}$$

$$= -.07564947 + .09624555 + .08296228$$
$$- .07481066 - .08309485 - .09716943$$

$\hat{u}_1 \cdot \hat{u}_2 \times \hat{u}_3 = -.00021764$

$$\hat{u}_1 \times \hat{u}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ .48220704 & .85248609 & .20185104 \\ .57787890 & .79409651 & .18832606 \end{vmatrix}$$

$\hat{u}_1 \times \hat{u}_3 = .00025614\,\hat{i} + .02583331\,\hat{j} - .10971480\,\hat{k}$

$a_1\vec{R}_1 - \vec{R}_2 + a_3\vec{R}_3 = .01538116\,\hat{i} + .01702452\,\hat{j} + .00737938\,\hat{k}$

$$A = \frac{(a_1\vec{R}_1 - \vec{R}_2 + a_3\vec{R}_3)\cdot(\hat{u}_1 \times \hat{u}_3)}{\hat{u}_1 \cdot \hat{u}_2 \times \hat{u}_3} = \frac{-.00036589}{-.00021764} = 1.68118121$$

$b_1\vec{R}_1 + b_3\vec{R}_3 = -.01461321\,\hat{i} - .01608799\,\hat{j} - .00697663\,\hat{k}$

$$B = \frac{(b_1\vec{R}_1 + b_3\vec{R}_3)\cdot(\hat{u}_1 \times \hat{u}_3)}{\hat{u}_1 \cdot \hat{u}_2 \times \hat{u}_3} = \frac{.00034609}{-.00021764} = -1.59021674$$

$$\rho_2 = 1.68118121 - \frac{1.59021674}{r_2^3}$$

from $r_2^2 = \rho_2^2 + R_2^2 - 2\rho_2(\hat{u}_2 \cdot \vec{R}_2)$

$r_2^2 = \rho_2^2 + .98011651 + 1.92591860\,\rho_2$

Try $r_2 = 2.4 \Rightarrow \rho_2 = 1.56614817 \rightarrow r_2 = 2.53952958$

$\Rightarrow \rho_2 = 1.58408629 \Rightarrow r_2 = 2.55739460 \Rightarrow \rho_2 = 1.58610692$

$\Rightarrow r_2 = 2.55940706 \Rightarrow \rho_2 = 1.58633101 \Rightarrow r_2 = 2.55963025$

$\Rightarrow \rho_2 = 1.58635582 \Rightarrow r_2 = 2.55965496 \Rightarrow \rho_2 = 1.58635857$

$\Rightarrow r_2 = 2.55965769 \Rightarrow \rho_2 = 1.58635887 \Rightarrow r_2 = 2.55965799$

$\Rightarrow \rho_2 = 1.58635891 \Rightarrow r_2 = 2.55965803 \Rightarrow \rho_2 = 1.58635891$ ✓

$\Rightarrow r_2 = 2.55965803$ ✓

$$c_1 = a_1 + \frac{b_1}{r_2^3} = .61538462 + \frac{.01274699}{(2.55965803)^3}$$

$$c_1 = .61614470$$

$$c_3 = a_3 + b_3/r_2^3 = .38461539 + \frac{.01092549}{(2.55965803)^3}$$

$$c_3 = .38526689$$

$$c_1\vec{R}_1 - \vec{R}_2 + c_3\vec{R}_3 = .01450979\,\hat{\imath} + .01606522\,\hat{\jmath} + .00696338\,\hat{k}$$

$$(c_1\vec{R}_1 - \vec{R}_2 + c_3\vec{R}_3)\cdot\hat{u}_2\times\hat{u}_3 = \begin{vmatrix} .01450979 & .01606522 & .00696338 \\ .51757867 & .83303247 & .19536943 \\ .57787890 & .79409651 & .18832606 \end{vmatrix}$$

$$= .00227632 + .00181376 + .00286200 - .00225108 - .00156593$$
$$- .00335211 = -.00021705$$

$$\rho_1 = \frac{(c_1\vec{R}_1 - \vec{R}_2 + c_3\vec{R}_3)\cdot\hat{u}_2\times\hat{u}_3}{c_1\,\hat{u}_1\cdot\hat{u}_2\times\hat{u}_3} = 1.61856756$$

$$(c_1\vec{R}_1 - \vec{R}_2 + c_3\vec{R}_3)\cdot\hat{u}_1\times\hat{u}_2 = \begin{vmatrix} .01450979 & .01606522 & .00696338 \\ .48220704 & .85248609 & .20185104 \\ .51757867 & .83303247 & .19536943 \end{vmatrix}$$

$$= .00241660 + .00167839 + .00279715 - .00243980$$
$$- .00151348 - .00307244 = -.00013358$$

$$\rho_3 = \frac{(c_1\vec{R}_1 - \vec{R}_2 + c_3\vec{R}_3)\cdot\hat{u}_1\times\hat{u}_2}{c_3\,\hat{u}_1\cdot\hat{u}_2\times\hat{u}_3} = 1.59306009$$

$$\vec{r}_1 = \rho_1\hat{u}_1 - \vec{R}_1 = 1.5607323\,\hat{\imath} + 1.9424988\,\hat{\jmath} + .5707227\,\hat{k}$$

$$\vec{r}_2 = \rho_2\hat{u}_2 - \vec{R}_2 = 1.4809003\,\hat{\imath} + 1.9986228\,\hat{\jmath} + .6035647\,\hat{k}$$

$$\vec{r}_3 = \rho_3\hat{u}_3 - \vec{R}_3 = 1.3477791\,\hat{\imath} + 2.0810209\,\hat{\jmath} + .6538678\,\hat{k}$$